THE UNIVERSITY OF AUCKLAND

SECOND SEMESTER, 2011
Campus: City

SOFTWARE ENGINEERING

Software Engineering Theory

(Time allowed: TWO hours)

NOTE:
- There are eleven questions. Answer all of them.
- The total number of marks is 45.
- Calculators are not permitted.
- Show all your work and keep your proofs short.

1. On the set $A = \{2, 3, 4, 5, 6\}$, consider the following relations and answer the questions about each of them. Keep your answers short.

(a) $R_1 = \{(2,2), (3,3), (4,4), (5,5), (6,6)\}$. Is this relation symmetric? Is this relation transitive? Is this relation antisymmetric? (3 marks)

(b) $R_2 = \{(x, y) \mid x \leq y\}$. Is this relation transitive? (1 mark)

(c) $R_3 = \{(x, y) \mid y \text{ is a factor of } x\}$. This is a partial order on A. List all maximal and minimal elements of this partial order. Draw a Hasse diagram of the partial order. (3 marks)

2. On the set $A = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$ consider the following function $f : A \to A$. For all $n \in A$, if $n \neq 4$ then $f(n) = n + 1$; If $n = 4$ then $f(n) = -4$.

(a) Is the function surjective (onto)? Explain your answer. (2 marks)

(b) Is the function injective? Explain your answer. (2 marks)

3. Consider the following statement:

If n is a natural number then n can be written in the form $2a + 3b$ for some integers a and b.

(a) Write down the hypothesis and conclusion of this statement. (2 marks)

(b) Prove the statement **using induction**:

(i) State the basis of the induction. (1 mark)

(ii) State the inductive hypothesis. (1 mark)

(iii) Provide the inductive step of the proof. (2 marks)

4. Consider the directed graph defined as follows. The vertex set is

$$\{-4, -3, -2, -1, 0, 1, 2, 3, 4, 6, 7, 8\}.$$

The edges set $E = \{(x, y) \mid |x| + 1 = y\} \cup \{(4, 0), (3, -2), (-2, -1), (-4, 6), (8, -4)\}$. Recall that $|x|$ refers to the absolute value of x. Do the following:

(a) Draw this graph. (1 mark)

(b) Write down the in-degrees of vertices 2 and -4. (1 mark)

(c) How many strongly connected components does this graph have? (1 mark)

(d) Write down all the strongly connected components of the graph. (1 mark)

(e) Write down an adjacency list representation of this graph? (1 mark)

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5. Consider the $K_{n,m}$ he following undirected graph, where $n, m \geq 1$: Its vertex set is

$$V = \{a_1, \ldots, a_n, b_1, \ldots, b_m\}.$$

Its edge set is $E = \{\{a_i, b_j\} \mid i = 1, \ldots, n;\ j = 1, \ldots, m\}$.

(a) Draw $K_{4,5}$. (1 mark)

(b) How many edges does $K_{n,m}$ have? (1 mark)

(c) Which of these graphs $K_{n,m}$ have Euler circuits? (1 mark)

(d) Which of these graphs $K_{n,m}$ have Hamiltonian circuits? Explain your answer. Keep your explanation short. (3 marks)

6. Consider the following formula ϕ of propositional logic:

$$((p \wedge (s \rightarrow p)) \wedge (\neg q \vee s)).$$

(a) Convert the following formula into a DNF form. (2 marks)

(b) Is ϕ satisfiable? Explain your answer. (1 mark)

7. Consider the following language L over the binary alphabet $\{a, b\}$:

$$L = \{aub \mid |u| \text{ is an odd number}\},$$

where $|u|$ indicates the length of strings u.

(a) Draw a transition diagram of a NFA recognizing L. (2 marks)

(b) Write down a regular expression for L. (1 mark).

8. Consider the following NFA over the alphabet $\{a, b, c\}$:

- The states are 0, 1, and 2.
- 0 is the initial state, and 2 is the accepting state.
- The transition function T is this: $T(0, a) = 0$, $T(0, b) = T(0, c) = \{0, 1\}$, and $T(1, a) = T(1, b) = T(1, c) = \{2\}$.

Do the following:

(a) Draw transition diagram of the automaton. (1 mark).

(b) Describe the language recognized by the automaton. (1 mark)

(c) Determinize the automaton by applying the subset construction. Here you can draw a transition diagram of the deterministic automaton. (2 marks)

9. Let L_1 and L_2 be languages recognized by NFA with n and m states, respectively. Show that $L_1 \cup L_2$ can be recognized by an NFA with $n + m$ states. (3 marks)

10. Consider the directed weighted graph drawn below.

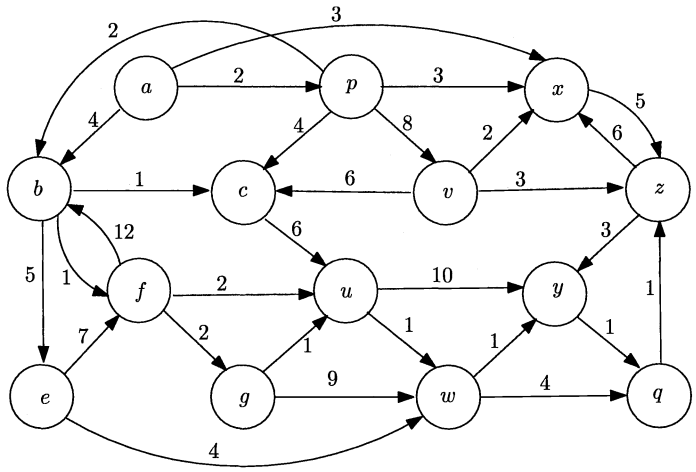

Figure 1: An example of a weighted graph G

Run the first 6 steps of Dijkstra's algorithm on this graph starting at node a. You need to say what the values of d (current distance estimate) and F (processed vertices) are at each step. (2 marks)

11. Explain why each of the the following equalities is true:

(a) $C(n,k) = C(n,n-k)$.

(b) $C(n,0) + C(n,1) + \ldots + C(n,n) = 2^n$.

Keep your explanation short. (2 marks)